Collinearity of points on Poincaré unit disk and Riemann sphere

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Abstract. We study certain points significant for the hyperbolic geometry of the unit disk. We give explicit formulas for the intersection points of the Euclidean lines and the stereographic projections of great circles of the Riemann sphere passing through these points. We prove several results related to collinearity of these intersection points, offer new ways to find the hyperbolic midpoint, and represent a formula for the chordal midpoint. The proofs utilize Gröbner bases from computer algebra for the solution of polynomial equations.

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