Rings whose nil-clean and clean elements are uniquely nil-clean

By JIAN CUI (Wuhu), PETER DANCHEV (Sofia) and DANYA JIN (Wuhu)

Abstract. We consider and study those rings in which each nil-clean or clean element is uniquely nil-clean. We establish that those rings are abelian. More precisely, it is shown that the classes of abelian rings and the rings in which nil-clean elements are uniquely nil-clean do coincide. Moreover, we prove that the rings in which clean elements are uniquely nil-clean coincide with the subclass of abelian rings consisting of only unipotent units. In particular, we obtain in the semi-potent case a complete characterization only in terms of the former ring and its divisions. Likewise, some extension properties and group rings for such kinds of rings are also considered.

JIAN CUI &
DANYA JIN
SCHOOL OF MATHEMATICS
AND STATISTICS
ANHUI NORMAL UNIVERSITY
WUHU 241002
CHINA
AND

JIAN CUI
KEY LABORATORY OF FINANCIAL
MATHEMATICS (PUTIAN UNIVERSITY)
FUJIAN PROVINCE UNIVERSITY
FUJIAN PUTIAN 351100
CHINA

PETER DANCHEV
INSTITUTE OF MATHEMATICS
AND INFORMATICS
BULGARIAN ACADEMY OF SCIENCES
"ACAD. G. BONCHEV" STR., BL. 8
1113 SOFIA
BULGARIA