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Dedekind sums and mean square value of $L(1, \chi)$ over subgroups

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Abstract. An explicit formula for the quadratic mean value at $s = 1$ of the Dirichlet L-functions associated with the odd Dirichlet characters modulo $f > 2$ is known. Here, we present a situation where we could prove an explicit formula for the quadratic mean value at $s = 1$ of the Dirichlet L-functions, associated with the odd Dirichlet characters modulo not necessarily prime moduli $f > 2$ that are trivial on a subgroup H of the multiplicative group $(\mathbb{Z}/f\mathbb{Z})^*$. This explicit formula involves summation $S(H, f)$ of Dedekind sums $s(h, f)$ over the $h \in H$. A result on some cancelation of the denominators of the $s(h, f)$'s when computing $S(H, f)$ is known. Here, we prove that for some explicit families of f's and H's, this known result on cancelation of denominators is the best result one can expect. Finally, we surprisingly prove that for p a prime, $m \geq 2$ and $1 \leq n \leq m/2$, the values of the Dedekind sums $s(h, p^m)$ do not depend on h as h runs over the elements of order p^n of the multiplicative cyclic group $(\mathbb{Z}/p^m\mathbb{Z})^*$.

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