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Weakly affine functions

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Abstract. Let $\emptyset \neq D \subseteq \mathbb{R}^n$ be a convex set. We say that the function $f : D \longrightarrow \mathbb{R}$ is weakly k-affine if, for any affinely independent system of $k+1$ points $x_0, x_1, \ldots, x_k \in D$, there exist $\lambda_0, \lambda_1, \ldots, \lambda_k \in]0,1[$ such that $\lambda_0 + \lambda_1 + \cdots + \lambda_k = 1$ and

$$f(\lambda_0 x_0 + \lambda_1 x_1 + \cdots + \lambda_k x_k) = \lambda_0 f(x_0) + \lambda_1 f(x_1) + \cdots + \lambda_k f(x_k).$$

Our main result is that any continuous, weakly 2-affine function is necessarily affine. It is a well-known result that a continuous, weakly 1-affine function is affine. However, in the paper, we will show that if continuity is replaced by several weaker regularity conditions, then this implication fails to hold. We also introduce a new concept of generalized convexity, namely the class of weakly k-convex sets, which turns out to be naturally related to weakly k-affine functions. We present results concerning a subclass of continuous, weakly k-affine functions for $k \geq 3$, as well.

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