

**On the (H, L) typeness of the maximal function
of Cesàro means of two-parameter
integrable functions on bounded Vilenkin groups**

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Abstract. In this paper we prove that the operator $\sigma^* f := \sup_{n \in \mathbb{N}^2} |\sigma_n f|$ from the Hardy space H to $L^1(G_m \times G_{\tilde{m}})$ is bounded, where the quotient of the coordinates of n is bounded. In other words σ^* is of type (H, L) .

1. Introduction

Let us briefly introduce the necessary definitions and notations. Let $m := (m_0, m_1, \dots)$ denote a sequence of positive integers not less than 2. Denote by $Z_{m_j} := \{0, 1, \dots, m_j - 1\}$ the additive group of integers modulo m_j ($j \in \mathbb{N}$). Z_{m_j} is endowed by the discrete topology, i.e. every subset of Z_{m_j} is open.

Define the group G_m as the cartesian product of the discrete cyclic groups Z_{m_j} ,

$$G_m := \prod_{j=0}^{\infty} Z_{m_j}.$$

The elements of G_m can be represented by sequences $x := (x_0, x_1, \dots, x_j, \dots)$ ($x_j \in Z_{m_j}$, $j \in \mathbb{N}$). The group operation on G_m is the coordinate-wise addition, the inverse operation is $-$. The topology on G_m is the product topology, and G_m is a compact Abelian

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group. A base $I_n(x)$, $n = 0, 1, \dots$ for the topology of G_m can be given in the following way. Set

$$I_0(x) := G_m, \quad I_n(x) := \{y = (y_i, i \in \mathbb{N}) \in G_m : y_i = x_i \text{ for } i < n\}$$

for $x \in G_m$, $n \in \mathbb{P} := \mathbb{N} \setminus \{0\}$. The element $(0, 0, \dots)$ is the nullelement of G_m it will be denoted by 0. Put $I_n = I_n(0)$ ($n \in \mathbb{N}$), and observe that I_n is a subgroup of G_m ($n \in \mathbb{N}$). The direct product μ of the measures

$$\mu_k(\{j\}) := \frac{1}{m_k} \quad (j \in Z_{m_k}, \quad k \in \mathbb{N})$$

is the Haar measure on G_m with $\mu(G_m) = 1$.

If $M_0 = 1$, $M_{k+1} = m_k M_k$ ($k \in \mathbb{N}$), then every $n \in \mathbb{N}$ can be uniquely expressed as $n = \sum_{j=0}^{\infty} n_j M_j$, where $n_j \in Z_{m_j}$ ($j \in \mathbb{N}$) and only a finitely many of n_j differ from zero.

The group G_m is metrizable. Define the distance between the elements $x \in G_m$ and $y \in G_m$ by

$$d(x, y) := \sum_{k=0}^{\infty} \frac{|x_k - y_k|}{M_{k+1}}.$$

The topology induced by this metric coincides with that of G_m . Denote by $L^p(G_m)$ the usual Lebesgue space with the corresponding norm.

$$\|f\|_p := \left(\int_{G_m} |f|^p \right)^{\frac{1}{p}} \quad (f \in L^p(G_m)), \quad 1 \leq p < \infty,$$

$$\|f\|_{\infty} := \inf\{r : r > 0, \quad \mu(|f| > r) = 0\} \quad (f \in L^{\infty}(G_m)).$$

Let $\hat{G}_m := \{\psi_n : n \in \mathbb{N}\}$ denote the character group of G_m . We enumerate the elements as follows. Denote on G_m the *generalized Rademacher functions* by

$$r_k(x) := \exp \frac{2\pi i x_k}{m_k} \quad (i^2 := -1, \quad x \in G_m, \quad k \in \mathbb{N}).$$

It is known that the functions

$$\psi_n(x) := \prod_{k=0}^{\infty} r_k^{n_k}(x) \quad (n \in \mathbb{N})$$

on G_m are elements of the character group of G_m , and all the elements of the character group are of this form.

The system $(\psi_n : n \in \mathbb{N})$ is called a *Vilenkin system* and G_m a *Vilenkin group*.

Let $\mathcal{A}_n$ be the σ -algebra generated by the cosets $I_n(z)$, where $n \in \mathbb{N}$, $z \in G_m$. Let α_j^k, α_n ($k, j, n \in \mathbb{N}$) be functions satisfying the following conditions:

- (i) $\alpha_j^k : G_m \rightarrow \mathbb{C}$ is $\mathcal{A}_j$ -measurable ($k, j \in \mathbb{N}$),
- (ii) $|\alpha_j^k| := \alpha_0^k := \alpha_j^0 := \alpha_j^k(0) := 1$ ($k, j \in \mathbb{N}$),
- (iii) $\alpha_n := \prod_{j=0}^{\infty} \alpha_j^{n^{(j)}} \quad \left(n \in \mathbb{N}, \quad n^{(j)} := \sum_{k=j}^{\infty} n_k M_k \right)$.

Let $\chi_n = \psi_n \alpha_n$ ($n \in \mathbb{N}$). A function system $\{\chi_n : n \in \mathbb{N}\}$ of this type is called a $\psi\alpha$ (*Vilenkin-like*) system on the Vilenkin group G_m . [Gát1, 2]

In [Gát2] it is proved that a Vilenkin-like system is orthonormal and complete in $L^1(G_m)$.

We mention some examples.

1. If $\alpha_j^k = 1$ for each $k, j \in \mathbb{N}$, then we obtain the “ordinary” Vilenkin systems [Vil].

2. If $m_j = 2$ for all $j \in \mathbb{N}$ and $\alpha_j^{n^{(j)}} = (\beta_j)^{n_j}$, where

$$\beta_j(x) = \exp \left(2\pi i \left(\frac{x_{j-1}}{2^2} + \cdots + \frac{x_0}{2^{j+1}} \right) \right) \quad (n, j \in \mathbb{N}, \quad x \in G_m),$$

then we get the character system of the group of 2-adic integers (see e.g. [Tai, HR, Gát7]).

The (about) 25 years old conjecture of M. H. TAIBLESON [Tai], namely that $\sigma_n f \rightarrow f$ ($f \in L^1$) a.e. (with respect to this system) was proved by GÁT [Gát7]. In [BG] the two-dimensional version of this question was also solved.

3. If

$$t_n(x) := \exp \left(2\pi i \left(\sum_{j=0}^{\infty} \frac{n_j}{M_{j+1}} \right) \sum_{j=0}^{\infty} x_j M_j \right) \quad (x \in G_m, n \in \mathbb{N}),$$

then we have a Vilenkin-like system which is a useful tool in the approximation theory of limit periodic, almost even arithmetical functions (cf. [Gát3]).

Define the Fourier coefficients, the partial sums of the Fourier series and the Fejér means with respect to the Vilenkin-like system χ in the following way:

$$\begin{aligned} \widehat{f}(n) &:= \int_{G_m} f \bar{\chi}_n, \quad S_n f := \sum_{k=0}^{n-1} \widehat{f}^\chi(k) \chi_k, \quad \sigma_n f := \frac{1}{n} \sum_{k=0}^{n-1} S_k^\chi f \\ &\left(n \in \mathbb{P}, \widehat{f}^\chi(0) := \int_{G_m} f, f \in L^1(G_m) \right). \end{aligned}$$

For more details about Vilenkin and Vilenkin-like systems see [SWS, AVD, Vil, Gát1-Gát5].

For $(n_1, n_2) = n \in \mathbb{N}^2$ set $\vee n := \max(n_1, n_2)$, $\wedge n := \min(n_1, n_2)$. Let $\tilde{m}$ also a sequence of integers with the same properties as m . In the sequel the **boundedness** of the **Vilenkin groups** $G_m, G_{\tilde{m}}$ is supposed. Denote the $L^p(G_m \times G_{\tilde{m}})$ norm of any function f by $\|f\|_p$ ($1 \leq p \leq \infty$).

Let $\tilde{I}$ denote the n -th interval generated by $\tilde{m}$. Define $\tilde{n} = \tilde{n}(n) := \min(l \in \mathbb{N} : M_n \leq \tilde{M}_l)$ ($n \in \mathbb{N}$). Then there exists a constant c for which $M_n \leq \tilde{M}_{\tilde{n}} < cM_n$ for all $n \in \mathbb{N}$ (c does not depend on n , but depends on $\max_{j \in \mathbb{N}} m_j$ and $\max_{j \in \mathbb{N}} \tilde{m}_j$).

So, from now on we are going to use cM_n instead of $\tilde{M}_{\tilde{n}}$ (in some inequalities below).

The Kronecker product $\{\chi_{n,m} : n, m \in \mathbb{N}\}$ of two Vilenkin-like systems $\{\chi_n : n \in \mathbb{N}\}$ and $\{\tilde{\chi}_n : n \in \mathbb{N}\}$ is said to be a two-dimensional (or double) Vilenkin-like system. Thus

$$\chi_{n,m}(x, y) := \chi_n(x) \tilde{\chi}_m(y),$$

where $x \in G_m, y \in G_{\tilde{m}}$.

If $f \in L^1(G_m \times G_{\tilde{m}})$ then the (n, k) -th Fourier coefficients, the (n, k) -th partial sum of Fourier series and the Fejér means of order (n, k) of double Fourier series are the following

$$\begin{aligned} \widehat{f}(n, k) &:= \int_{G_m \times G_{\tilde{m}}} f \bar{\chi}_{n,k}, \quad S_{n,k} f := \sum_{j=0}^{n-1} \sum_{l=0}^{k-1} \widehat{f}(j, l) \chi_{j,l}, \\ \sigma_{n,k} f &:= \frac{1}{nk} \sum_{j=0}^{n-1} \sum_{l=0}^{k-1} S_{j,l} f. \end{aligned}$$

For each $f \in L^1(G_m \times G_{\tilde{m}})$ let the maximal function and the diagonal maximal function be defined by

$$f^\star := \sup_{n_1, n_2 \in \mathbb{N}} \left| S_{M_{n_1}, \tilde{M}_{n_2}} f \right| \quad f^\circ := \sup_{n \in \mathbb{N}} \left| S_{M_n, \tilde{M}_n} f \right|$$

and

$$\|f\|_H := \|f^\star\|_1, \quad \|f\|_{H^\circ} := \|f^\circ\|_1.$$

Define the Hardy space H to be the collection of functions $f \in L_1(G_m \times G_{\tilde{m}})$ such that $\|f\|_H < \infty$ i.e.

$$H := \{f \in L_1(G_m \times G_{\tilde{m}}) : \|f\|_H < \infty\},$$

and the diagonal Hardy space H° as

$$H^\circ := \{f \in L_1(G_m \times G_{\tilde{m}}) : \|f\|_{H^\circ} < \infty\}.$$

Since $\|f\|_{H^\circ} \leq \|f\|_H$, thus $H^\circ \subset H$.

The atomic decomposition is a useful characterisation of Hardy spaces. In order to show this, we introduce the concept of an atom.

We say that $a \in L^\infty(G_m \times G_{\tilde{m}})$ is a H° atom if

- (i) there exist $x = (x_1, x_2)$ and $k \in \mathbb{N}$ such that $\text{supp } a \subset I_k(x_1) \times \tilde{I}_k(x_2)$,
- (ii) $\|a\|_\infty \leq M_k \tilde{M}_k$,
- (iii) $(E_k a)(y) := (E_{k, \tilde{k}} a)(y) = 0$ for all $y \in G_m \times G_{\tilde{m}}$ $(E_{r,s} f)(y) = M_r \tilde{M}_s \int_{I_r(y_1) \times \tilde{I}_s(y_2)} f$.

We will use the following decomposition theorem (similar and others can be found in [Wei1]).

Theorem 1. *A function $f \in L^1(G_m \times G_{\tilde{m}})$ is in the diagonal Hardy space H° if and only if there exist a sequence $(a_k : k \in \mathbb{N})$ of H° -atoms and a sequence $(\lambda_k : k \in \mathbb{N})$ of real numbers such that*

$$(1) \quad f = \sum_{k=0}^{\infty} \lambda_k a_k$$

and

$$(2) \quad \sum_{k=0}^{\infty} |\lambda_k| < \infty.$$

Moreover, $\|f\|_{H^\circ}$ and $\inf(\sum_{k=0}^\infty |\lambda_k|)$ are equivalent norms, where the infimum is taken over all decompositions of f .

To prove Theorem 1 recall the first steps of the decomposition algorithm of Calderon–Zygmund (for the original version see [CZ], [SWS]). For any $f \in L^1(G_m \times G_{\tilde{m}})$ and $\lambda > 0$ set

$$\Omega_0 := \begin{cases} I_0(x_1^0) \times \tilde{I}_0(x_2^0) & \text{if } |E_0 f(\mathbf{x}^0)| > \lambda \\ \emptyset & \text{otherwise} \end{cases}$$

and

$$\Omega_k := \begin{cases} \bigcup_i I_k(x_1^i) \times \tilde{I}_k(x_2^i) & \text{if } |E_k f(\mathbf{x}^i)| > \lambda, \\ \left(I_k(x_1^i) \times \tilde{I}_k(x_2^i) \right) \cap \left(\bigcup_{j=0}^{k-1} \Omega_j \right) = \emptyset & \\ \emptyset & \text{otherwise,} \end{cases}$$

where $\mathbf{x}^i = (x_1^i, x_2^i) \in G_m \times G_{\tilde{m}}$, and $I_k(x_1^i) \times \tilde{I}_k(x_2^i)$ denotes the i th rectangle defined in the course of procedure (start the counting from zero).

For the elements of Ω_k we introduce the following notation $I_k^i := I_k(x_1^i) \times \tilde{I}_k(x_2^i)$, where $i \in \mathbb{N}$. Let

$$\begin{aligned} f_k^i &:= f 1_{I_k^i} - \left(M_k \tilde{M}_k \int_{I_k^i} f \right) 1_{I_k^i} \\ f_\lambda &:= f 1_{\overline{\Omega_k}} + \sum_{i,k} \left(M_k \tilde{M}_k \int_{I_k^i} f \right) 1_{I_k^i}, \end{aligned}$$

where $1_{I_k^i}$ is the characteristic function of I_k^i . Thus $f = f_\lambda + \sum_{i,k} f_k^i$ and the number of elements of Ω_k is finite. These facts imply a modified version of the Calderon–Zygmund decomposition lemma, (see [CZ], [SW]):

Lemma 2. *Let $f \in L^1(G_m \times G_{\tilde{m}})$, $\lambda > \|f\|_1$. Then $f = f_\lambda + \sum_{n=1}^\infty f_n$, where $\|f_\lambda\|_\infty < c\lambda$, $\text{supp } f_n \subseteq I_{k_n}^n$ for some rectangles $(x_1^n \in G_m, x_2^n \in G_{\tilde{m}}, k_n \in \mathbb{N})$, $\int_{I_{k_n}^n} f_n = 0$, $\|f_n\|_1 \leq c\lambda\mu(I_{k_n}^n)$ ($n \in \mathbb{P}$). The rectangles $I_{k_n}^n$ are disjoint, furthermore $\mu(\Omega) = \mu\left(\bigcup_{n \in \mathbb{P}} I_{k_n}^n\right) \leq c \frac{\|f\|_1}{\lambda}$.*

PROOF of Theorem 1. Throughout this paper c will denote a constant which may vary at different occurrences and may depend only on β (defined later), $\sup m_n$ and $\sup \tilde{m}_n$.

For a H° -atom a we have

$$\|a\|_{H^\circ} = \left\| \sup_{n \in \mathbb{N}} |E_n a| \right\|_1 \leq \left\| \sup_{n \in \mathbb{N}} E_n \left(M_k \widetilde{M}_k 1_{I_k(x_1) \times \tilde{I}_k(x_2)} \right) \right\|_1 \leq 1.$$

Consequently, if a function f in $L^1(G_m \times G_{\tilde{m}})$ is of the form $f = \sum_{i=0}^{\infty} \lambda_i a_i$ with (2) then

$$\|f\|_{H^\circ} \leq \sum_{i=0}^{\infty} |\lambda_i| \|a_i\|_{H^\circ} \leq \sum_{i=0}^{\infty} |\lambda_i| < \infty,$$

that is, f is in H° . The first part of the proof is complete.

Conversely, use the Calderon–Zygmund decomposition lemma with $\lambda = 2^q$ and $q \in \mathbb{Z}$. We have

$$f = f_{2^q} + \sum_{n=1}^{\infty} f_{2^q, n},$$

where $\|f_{2^q}\|_\infty \leq c2^q$, and for a fixed $q \in \mathbb{Z}$ the sets $\text{supp } f_{2^q, n} = I_{k_q, n}^n$ are pairwise disjoint. Moreover, from the decomposition algorithm we obtain $2^q < |E_q f_{2^q, i}| < c2^q$, and $|E_j f_{2^q, n}| \leq 2^q$ for any $j = 0, 1, \dots, q-1$. However, $f_{2^q} \rightarrow f$ and $f_{-2^q} \rightarrow 0$ as $q \rightarrow \infty$, thus

$$f = \sum_{q=-\infty}^{\infty} (f_{2^{q+1}} - f_{2^q}) = \sum_{q=-\infty}^{\infty} \left(\sum_{n=1}^{\infty} f_{2^q, n} - \sum_{j=1}^{\infty} f_{2^{q+1}, j} \right)$$

can be written. There exist for a $j \in \mathbb{P}$ and $n \in \mathbb{P}$ such that $\text{supp } f_{2^{q+1}, j} \subset \text{supp } f_{2^q, n}$. Let

$$b_{q, n} := f_{2^q, n} - \sum_{j: \text{supp } f_{2^{q+1}, j} \subset \text{supp } f_{2^q, n}} f_{2^{q+1}, j}.$$

For every element $\mathbf{x}$ of $\text{supp } f_{2^q, n}$ the equation $b_{q, n}(\mathbf{x}) = f_{2^{q+1}}(\mathbf{x}) - f_{2^q}(\mathbf{x})$ holds, consequently $|b_{q, n}| \leq c2^q$. Let $a_{q, n} := M_q \widetilde{M}_q \frac{1}{c2^q} b_{q, n}$, and $\lambda_{q, n} := c2^q \frac{1}{M_q \widetilde{M}_q}$, $q \in \mathbb{Z}$, $n \in \mathbb{P}$ then

$$f = \sum_{q=-\infty}^{\infty} \sum_{n=1}^{\infty} \lambda_{q, n} a_{q, n},$$

where the functions $a_{q,n}$ are H° -atoms with supports $I_{k_{q,n}}^n$.

$$\begin{aligned} \sum_{q=-\infty}^{\infty} \sum_{n=1}^{\infty} |\lambda_{q,n}| &= c \sum_{q=-\infty}^{\infty} 2^q \sum_{n=1}^{\infty} \mu(I_{k_{q,n}}^n) \\ &\leq c \sum_{q=-\infty}^{\infty} 2^q \mu(f^\circ > 2^q) \leq c \|f^\circ\|_1 = c \|f\|_{H^\circ}. \end{aligned}$$

This completes the proof. $\square$

2. The (H, L) typeness of the maximal operator

Define the maximal operator of Cesàro means of two parameter integrable functions on bounded Vilenkin groups as follows

$$\sigma^* f := \sup_{\substack{(n_1, n_2) \in \mathbb{N}^2 \\ \beta^{-1} \leq \frac{n_1}{n_2} \leq \beta}} |\sigma_n f| \quad (n = (n_1, n_2)),$$

where $\beta > 1$ is some fixed parameter.

In order to prove the main theorem we need the corollary of the following lemma (see [BG], and for the Walsh case see [Gát6]).

Lemma 3. *Let $A, k \in \mathbb{N}$ be fixed, $A > k - c$. Set $\text{supp } f \subseteq I_k(x_1) \times \tilde{I}_k(x_2)$, $f \in L^1(G_m \times G_{\tilde{m}})$ ($(x_1, x_2) \in G_m \times G_{\tilde{m}}$). Then we have*

$$\begin{aligned} \int_{(G_m \times G_{\tilde{m}}) \setminus I_k(x_1) \times \tilde{I}_k(x_2)} \sup \{ |\sigma_n f| : \mathbf{n} \in \mathbb{P}^2, \wedge n \geq M_A, \beta^{-1} \leq n_1/n_2 \leq \beta \} \\ \leq c \left(\frac{M_k}{M_A} \right)^{\frac{1}{2}} \|f\|_1. \end{aligned}$$

Corollary 4. *Suppose that $\text{supp } f \subseteq I_k(x_1) \times \tilde{I}_k(x_2)$ and $f \in L^1(G_m \times G_{\tilde{m}})$ ($k \in \mathbb{N}$, $(x_1, x_2) \in G_m \times G_{\tilde{m}}$). Then we have*

$$\int_{(G_m \times G_{\tilde{m}}) \setminus I_k(x_1) \times \tilde{I}_k(x_2)} \sigma^* f \leq c \|f\|_1.$$

Theorem 5. *The operator σ^* is of type (H°, L) i.e.*

$$\|\sigma^* f\|_1 \leq c \|f\|_{H^\circ} \quad (f \in H).$$

PROOF of Theorem 5. Recall that the operator σ^* from $L^p(G_m \times G_{\tilde{m}})$ to $L^p(G_m \times G_{\tilde{m}})$ is bounded for $p > 1$, (because of the weak $(1,1)$ and (∞, ∞) typeness of the operator σ^* (see [BG])). Suppose that a is an H° -atom with support $I_k(x_1) \times \tilde{I}_{\tilde{k}}(x_2)$. Using Corollary 4, Hölder's inequality and the boundedness of σ^* , we get

$$\begin{aligned} \int_{G_m \times G_{\tilde{m}}} \sigma^* a &= \int_{I_k(x_1) \times \tilde{I}_{\tilde{k}}(x_2)} \sigma^* a + \int_{(G_m \times G_{\tilde{m}}) \setminus (I_k(x_1) \times \tilde{I}_{\tilde{k}}(x_2))} \sigma^* a \\ &\leq \left(\int_{G_m \times G_{\tilde{m}}} (\sigma^* a)^p \right)^{\frac{1}{p}} \mu(I_k(x_1) \times \tilde{I}_{\tilde{k}}(x_2))^{1-\frac{1}{p}} + c \|a\|_1 \\ &\leq c_p \left(\int_{I_k(x_1) \times \tilde{I}_{\tilde{k}}(x_2)} |a|^p \right)^{\frac{1}{p}} \mu(I_k(x_1) \times \tilde{I}_{\tilde{k}}(x_2))^{1-\frac{1}{p}} + c \\ &\leq c_p \left((M_k \widetilde{M_{\tilde{k}}})^p \frac{1}{M_k \widetilde{M_{\tilde{k}}}} \right)^{\frac{1}{p}} \left(\frac{1}{M_k \widetilde{M_{\tilde{k}}}} \right)^{1-\frac{1}{p}} + c \leq c. \end{aligned}$$

Using the atomic decomposition of the function f and the σ -sublinearity of the operator σ^* ,

$$\|\sigma^* f\|_1 = \left\| \sigma^* \left(\sum_{i=1}^{\infty} \lambda_i a_i \right) \right\|_1 \leq \sum_{i=1}^{\infty} |\lambda_i| \|\sigma^* a_i\|_1 \leq c \sum_{i=1}^{\infty} |\lambda_i| \leq c \|f\|_{H^\circ}.$$

This completes the proof. $\square$

Since $\|\cdot\|_{H^\circ} \leq \|\cdot\|_H$, we have $H \subset H^\circ$. Consequently, we also have

Corollary 6. *The operator σ^* is of type (H, L) .*

Corollary 6 with respect to the ordinary Vilenkin systems (bounded case) can be found [Wei2].

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